A one-way quantum computer

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We present a scheme of quantum computation that consists entirely of one-qubit measurements on a particular class of entangled states, the cluster states. The measurements are used to imprint a quantum logic circuit on the state, thereby destroying its entanglement at the same time. Cluster states are thus one-way quantum computers and the measurements form the program.

Most of the current experiments are designed to implement sequences of highly controlled interactions between selected particles (qubits), thereby following models of a quantum computer as a (sequential) network of quantum logic gates [1,2].

Here we describe a different model of universal and scalable quantum computation, the one-way quantum computer ($QC_{\mathcal{C}}$). In our model, the entire resource for the quantum computation is provided initially in the form of a specific entangled state, a so-called cluster state $|\phi\rangle_{\mathcal{C}}$ [3] of a large number of qubits. Information is then written onto the cluster, processed, and read out from the cluster by one-particle measurements only. The entangled state of the cluster thereby serves as a universal "substrate" for any quantum computation. It provides in advance all entanglement that is involved in the subsequent quantum computation. In the process of computation, i.e. during the rounds of one-qubit measurements, all entanglement in the cluster state is destroyed such that the cluster state can be used only once. Therefore, we call this scheme the one-way quantum computer.

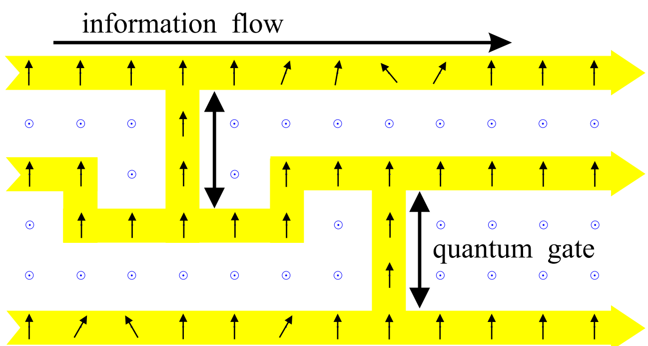

FIG. 1. Simulation of a quantum logic network by measuring two-state particles on a lattice. Before the measurements the qubits are in the cluster state $|\phi\rangle_{\mathcal{C}}$. Circles $\odot$ symbolize measurements of σ_z, vertical arrows are measurements of σ_x, while tilted arrows refer to measurements in the x-y-plane.

Cluster states can be created efficiently in any system with a quantum Ising-type interaction (at very low temperatures) between two-state particles in a lattice configuration. We consider two and three-dimensional arrays, or clusters, of qubits. To create a cluster state $|\phi\rangle_{\mathcal{C}}$ on the cluster $\mathcal{C}$ from a product state $\bigotimes_{a\in\mathcal{C}} |+\rangle_a$, (where $\sigma_x^{(a)}|\pm\rangle_a = \pm|\pm\rangle_a$), the Ising-interaction is switched on for an appropriately chosen finite time interval T, and is switched off afterwards. Since the Ising Hamiltonian acts uniformly on the lattice, an entire cluster of neighboring particles becomes entangled in a single step.

To process quantum information with the cluster $\mathcal{C}$ it suffices to measure its particles in a certain order and in a certain basis, as illustrated in Fig. 1. This figure shows, in a way, the physical and the logical layer of the $QC_{\mathcal{C}}$. The physical part is represented by the entangled cluster qubits and the measurements performed on them. The cluster qubits are displayed as dots "$\odot$" or as arrows "$\uparrow$", "$\nearrow$", depending on the respective measured observable (see caption). These measurements induce a quantum processing of logical qubits. The horizontal spatial axis on the cluster can be associated with the time axis of the implemented quantum circuit, i.e. with the direction of the "information flow". As will be explained, measurements of observables σ_z effectively remove the respective lattice qubit from the cluster. This property allows one to structure the cluster state on the lattice and imprint a network-like structure on it (displayed in Fig. 1 in gray underlay). More precisely, the σ_z-measurements project the cluster state $|\phi\rangle_{\mathcal{C}}$ into the tensor product $|\mu\rangle_{\mathcal{C}\backslash\mathcal{C}_N} \otimes |\tilde{\phi}\rangle_{\mathcal{C}_N}$. Therein, $|\mu\rangle_{\mathcal{C}\backslash\mathcal{C}_N}$ is a product state in the computational basis, and $|\tilde{\phi}\rangle_{\mathcal{C}_N}$ the state of the so far unmeasured qubits. It is again a cluster state on a network-shaped sub-cluster $\mathcal{C}_N$. On this sub-cluster quantum gates can be implemented via measurements of observables σ_x, σ_y, and linear combinations thereof. Measurements of σ_x and σ_y are used for "wires", i.e. to propagate logical quantum bits across the cluster, and for CNOT gates between two logical qubits. Observables of the form $\cos(\varphi)\,\sigma_x \pm \sin(\varphi)\,\sigma_y$ are measured to realize arbitrary rotations of logical qubits. For the cluster qubits of which a linear combination of σ_x and σ_y is measured, the measurement basis depends on the results of measurements at other cluster qubits. This introduces a temporal order among the measurements. The processing is finished once all qubits except a last one on each wire have been measured. The remaining unmeasured qubits form the quantum register which is now ready to be read out. At this point, the results of previous measurements determine in which basis these "output" qubits need to be measured for the final readout, or, if the readout measurements are in the σ_x-, σ_y- or σ_z-eigenbasis, how the readout measurements have to be interpreted.

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