Simultaneous cooling of axial vibrational modes in a linear ion-trap

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In order to use a collection of trapped ions for experiments where a well defined preparation of vibrational states is necessary (for example, for quantum information processing), all vibrational modes have to be cooled to ensure precise and repeatable manipulation of quantum states of internal and external degrees of freedom of the ions. A method for simultaneous sideband cooling of all axial vibrational modes is proposed. By application of a magnetic field gradient the absorption spectrum of each ion is modified such that sideband resonances of different vibrational modes coincide. The ion string is then irradiated with electromagnetic radiation of only a single frequency in the optical or microwave regime for sideband excitation. This new cooling scheme is described with an analytical treatment and investigated in detailed numerical studies.

I. INTRODUCTION

Atomic ions trapped in an electrodynamic cage allow for preparation and measurement of individual quantum systems, and represent an ideal system to investigate fundamental questions of quantum physics, for instance, related to decoherence [1, 2], the measurement process [3, 4], or multiparticle entanglement [5]. Also, trapped ions satisfy all criteria necessary for quantum computing. Two internal states of each ion represent one elementary quantum mechanical unit of information (a qubit). The quantized vibrational motion of the ions (the "bus-qubit") is used as means of communication between individual qubits to implement conditional quantum dynamics with two or more qubits [6]. The scheme for conditional dynamics proposed in [6] requires cooling to the ground state of vibrational motion in order to initialize the bus-qubit. Other schemes have been proposed that do not necessitate ground state cooling [7–9], yet still require low vibrational excitation of the ion string. Thus cooling of the ions' motional degrees of freedom is indispensable for quantum information processing (QIP), and whenever the well defined preparation of motional states of a collection of ions is desired.

Among recent important experimental contributions to laser cooling of trapped ions was the demonstration of ground state sideband cooling of 2 Be^+ ions [10]. The excitation of 5 out of 6 modes of two In^+ ions has been simultaneously reduced to mean vibrational quantum numbers around unity using bichromatic sideband cooling [11]. Two modes of a single ion have been cooled simultaneously close to their ground state making use of electromagnetically induced transparency [12], an effect taking advantage of light-induced atomic coherences. The cooling of all modes of two ions far into the Lamb-Dicke regime, robust against variations in experimental parameters, has been demonstrated [13]. Here too, the atomic absorption spectrum is optimally shaped for cooling by two light fields inducing atomic coherences. It is not required to resolve motional sidebands, and the laser parameters used for cooling are at the same time suitable for efficient state selective detection of internal ionic states.

It is necessary to cool not only the vibrational mode used as a bus-qubit for QIP, but also other modes that take no active role as bus-qubits ("spectator modes"), since the Rabi frequency for transitions between internal states of the ions depends on the motional state of all modes and, thus, the precision of quantum logic operations is severely limited, if spectator modes are thermally excited [14].

Sufficient cooling of the vibrational motion of two ions in a common trap potential has been demonstrated experimentally as was outlined above. This is deemed to be sufficient, even for a scalable quantum information processor, if only two ions *at a time* are used for quantum logic operations with additional ions stored in spatially separated regions [15].

If more than two ions reside in a common trap potential and shall be used simultaneously for quantum logic operations, however, the task of reducing the ions' motional thermal excitation becomes increasingly challenging with a growing number of ions. The number of axial vibrational modes in a linear ion trap is equal to the number N of ions and cooling them all sufficiently represents a severe obstacle on the way towards using a larger number of ions simultaneously for QIP.

Applying sideband cooling sequentially to each one of the modes will leave little time for quantum logic operations between cooling cycles. With many ions such sequential cooling might not work at all, since after having cooled the last one of N axial modes, the first one may already be considerably affected by heating. Therefore it is desirable to find new methods that allow for simultaneous and efficient cooling of multiple vibrational modes of many ions. Here, we propose a scheme where a magnetic field gradient applied to an electrodynamic ion trap is designed such that the first order red sidebands of all axial vibrational modes nearly coincide, and thus can all be sideband cooled by irradiating the ion string only with a single frequency. Either laser light or microwave radiation can be used for sideband excitation to implement this method for simultaneous cooling of many

vibrational modes [9, 16].

The remainder of this article is organized as follows: In section II the new cooling scheme is outlined. Cooling of axial vibrational modes of a linear string of ions is investigated either using an optical Raman transition or a microwave transition. In section III possible experimental implementations are discussed and the cooling scheme is studied under imperfect experimental conditions again with the aid of numerical studies. The analytical results presented in previous sections are derived in section IV before the is paper concluded in section V.

II. SIMULTANEOUS SIDEBAND COOLING OF AXIAL VIBRATIONAL MODES

A. Axial vibrational modes

We consider N singly ionized ions in a trap potential of cylindrical symmetry around the $z-$axis providing strong radial confinement such that the ions are aligned along this axis. For example, in a linear Paul trap [17] strong radial confinement is achieved by applying an rf voltage to the quadrupole electrodes while the weaker trapping potential along the $z-$axis is generated by a dc voltage. The radial and axial harmonic trapping potentials are characterized by the secular angular frequencies $\nu_r \gg \nu_z$ and the typical axial distance between neighboring ions is given by $\zeta_0 2N^{-0.57}$ with $\zeta_0 \equiv (e^2/(4\pi\epsilon_0 m\nu_1^2))^{1/3}$ [18].

Expanding the axial potential experienced by the ions in such a trap up to second order in the ions' displacements leads to a Hamiltonian that describes N uncoupled harmonic oscillators, the normal collective modes of vibration of the ion string, [19]

$$H = \sum_{n=1}^{N} \hbar\nu_n (a_n^\dagger a_n) \ . \tag{1}$$

Here, $a_n^\dagger$ and a_n are the creation and annihilation operators decribing the axial normal modes characterized by frequencies $\nu_1 < \nu_2 < \ldots < \nu_N$ and normal conjugate coordinates Q_α, P_α $(\alpha = 1, \ldots, N)$. The crystallized ions are localized at about the classical equilibrium positions $z_1, \ldots, z_N$ of the total potential comprised of the trap potential and mutual Coulomb repulsion. For N ions with same mass m and elementary charge e (the cas considered in this paper) ν_1 is equal to the axial secular frequency ν_z of the harmonic trap.

The local coordinate q_j (the deviation of ion j from its equilibrium position z_j; $j = 1, \ldots, N$) is obtained from $q_j = \sum_\alpha S_j^\alpha Q_\alpha$ where S_j^α are the elements of the unitary matrix $\hat{S}$ that transforms the dynamical matrix $\hat{A}$ such that $\hat{S^{-1}}\hat{A}\hat{S}$ is diagonal [19]. The normal modes are excited by displacing an ion from its equilibrium position z_j by an amount q_j. Thus, the coefficients S_j^α describe the strength with which a displacement q_j from z_j couples to the mode α.

Exciting a vibrational mode can be achieved through the mechanical recoil associated with the scattering of photons by the ions. In what follows we consider two internal states of each ion, $|0\rangle$ and $|1\rangle$, separated by $\hbar\omega_0$ in zero magnetic field. The linewidth of the $|0\rangle - |1\rangle$ resonance is indicated by γ. The ions interact individually with light [20, 21], and when their internal transition is driven well below saturation, the contributions of scattering from each ion to the excitation of the modes add up incoherently [21]. An important quantity, scaling this type of excitation, is the Lamb-Dicke parameter (LDP) [22]. If a photon is scattered by the ion at z_j and we consider the excitation of the mode at frequency ν_α, the LDP reads [23]

$$\eta_j^\alpha = S_j^\alpha \sqrt{\frac{\omega_R}{\nu_\alpha}} \tag{2}$$

Here, $\omega_R = \hbar k^2/2m$ is the recoil frequency, with $\vec{k}$ the wave vector of the light.

The LDP η_j^α gives a measure for how likely the absorption or emission of photons is, if the electromagnetic radiation driving the internal transition of an ion is tuned to a frequency $\omega_0 \pm \nu_\alpha$. Tuning the driving radiation to one of these sidebands makes it possible to (de-)excite internal and external degrees of freedom of the ions simultaneously. In particular, absorption of a photon on the red sideband $\omega_0 - \nu_\alpha$ results in the loss of a vibrational quantum of mode α. In the remainder of this article sideband resonances with excitation strength linear in η_j^α are considered. The presence of sideband resonances nonlinear in η_j^α does not affect the validity of the cooling scheme proposed here.

B. Sideband cooling of an ion chain

Assuming that the internal transition of an ion is driven under saturation by a laser with wave vector $\vec{k}$, and in the Lamb-Dicke regime, i.e. when the recoil frequency is much smaller than the trap frequency ν_z, the dynamics of the mode can be studied in perturbation theory at second order in the parameter ω_R/ν. A rate equation for the population $P_\alpha(n)$ of a n-th excitation of mode α can then be derived, and has the form [23]:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}P_\alpha(n) \ &= \ (n+1)\left[A_-^\alpha P_\alpha(n+1) - A_+^\alpha P_\alpha(n)\right] \\ &\quad + n\left[A_-^\alpha P_\alpha(n) - A_+^\alpha P_\alpha(n-1)\right]\end{aligned} \tag{3}$$

where A_+^α (A_-^α) characterizes the rate at which the mode is heated (cooled). For laser light at frequency ω_L, coupling with Rabi frequency Ω_j to an internal transition at resonance frequency ω_j and linewidth γ, the rates have the form

$$A_\pm^\alpha = \sum_{j=1}^{N} |\eta_j^\alpha|^2 \frac{\Omega_j^2}{2\gamma}\left[\frac{\gamma^2}{4(\delta_j \mp \nu_\alpha)^2 + \gamma^2} + \phi\frac{\gamma^2}{4\nu_\alpha^2 + \gamma^2}\right] \tag{4}$$

where the detuning $\delta_j \equiv \omega_L - \omega_j$. The coefficient ϕ emerges from the integral over the angles of photon emission, according to the pattern of emission of the given transition [22]. The average number of phonons of mode α at steady state is then given by

$$\langle n \rangle_\alpha = \frac{A_+^\alpha}{A_-^\alpha - A_+^\alpha} \ , \tag{5}$$

and the rate at which this steady-state value is reached is $W_\alpha = A_-^\alpha - A_+^\alpha$ [23].

For a single ion, if $\gamma \ll \nu$ and $\delta = \nu_z$, one gets $A_-^\alpha \gg A_+^\alpha$ (sideband cooling) [22]. In this limit, the average phonon number at steady state is very small, and the ion will be found in its motional ground state with high probability. In the presence of many modes at different (and incommensurate) frequencies, sideband cooling can be used to cool simultaneously all modes to the ground state. With the scheme outlined below this can be accomplished by shaping the excitation spectrum of the ions: for each mode α there is one ion j with the matching resonance frequency, that is, such that $\delta_j = \omega_L - \omega_j = \nu_\alpha$.

In general, whenever the motion of an ion can be sideband-cooled, so can the collective motion of an ion-chain. This is valid also for atomic configurations that do not have suitable transitions to directly apply sideband cooling, as long as a multi-photon process can be designed with an effective linewidth smaller than the trap frequency [24].

C. Shaping the spectrum of N ions

If a magnetic field whose magnitude varies as a function of z is applied to a linear ion trap, then appropriately chosen internal resonances of each ion will be Zeeman-shifted, and the ions' resonance frequencies ω_j are no longer degenerate. If the field gradient is designed such that all first order red sidebands $\omega_1 - \nu_1, \omega_2 - \nu_2, \ldots, \omega_N - \nu_N$ corresponding to the axial vibrational modes coincide, then all modes can be sideband cooled by irradiating the ion string with electromagnetic radiation only at a single frequency $\omega = \omega_1 - \nu_1$. If the ions, initially prepared in their lower internal state $|0\rangle$, are irradiated with electromagnetic radiation at this frequency, absorption can take place if the vibrational quantum number of any of the N modes is reduced by one. With appropriate recycling of the population from state $|1\rangle$ this leads to sideband cooling on all N modes simultaneously. This situation is illustrated in Fig. 1 for the case of 10 ions: The thickest horizontal lines indicate the ions' resonance frequencies $\omega_j - \omega_1$ relative to ω_1 in units of the secular axial frequency $\nu_1 = \nu_z$. The center of each line indicates an ion's axial position in units of ζ_0. The remaining lines show the frequencies of the accompanying red sideband resonances for each ion, $\omega_j - \omega_\alpha$. The resonances where $j = \alpha$ have been highlighted by medium thick lines. Here the field gradient along the z-axis is chosen such that $\omega_1 - \nu_1 = \omega_2 - \nu_2 = \ldots \omega_N - \nu_N$. Either laser light or microwave radiation [16] can be used for sideband excitation to implement this method for simultaneous cooling of all vibrational modes.

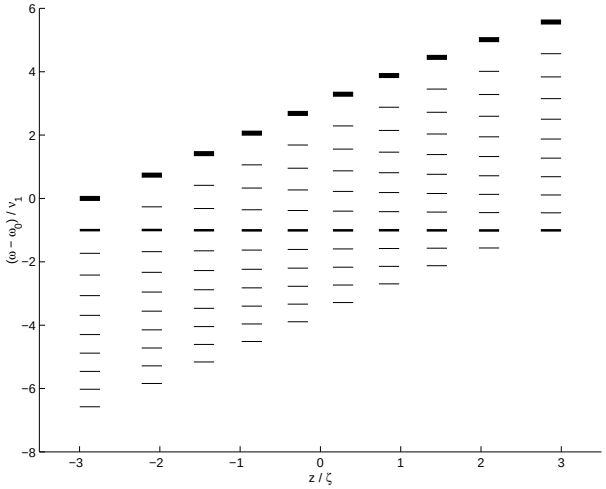

FIG. 1: With the absorption spectrum of a collection of ions suitably modified, sideband cooling can be applied to cool all axial vibrational modes simultaneously by irradiating the ion chain at a single frequency $\omega = \omega_1 - \nu_1$. Horizontal thick lines indicate the ionic resonances ω_j with $j = 1, \ldots, 10$. The other lines correspond to sideband resonances of each ion, $\omega_j - \nu_\alpha$. Medium thick lines emphasize the coinciding sideband resonances where $j = \alpha$. The center of each line shows the position of ion j in scaled coordinates.

As an example, we will now discuss simultaneous sideband cooling of a chain of $^{171}Yb^+$ions. The ions are crystallized along the axis of a linear trap, and a magnetic field $B(z)$ along the axis is applied that Zeeman-shifts the energy of the internal states. The value of the field along z is such that it shifts the red-sidebands of all modes into resonance along the chain, while at the same time its gradient is sufficiently weak to negligibly affect the frequencies of the normal modes [9].

The selective drive of the motional sidebands can be implemented on a magnetic dipole transition in $^{171}Yb^+$close to $\omega_0 = 12.6 \times 2\pi$ GHz between the hyperfine states $|0\rangle = |S_{1/2}, F = 0\rangle$ and $|1\rangle = |S_{1/2}, F = 1, m_F = 1\rangle$. Due to the magnetic field gradient, the degeneracy between the resonances of individual ions is lifted, and the transition frequency ω_j of ion j is proportional to $B(z_j)$ in the weak field limit ($\mu_B B/\hbar\omega_0 \ll 1$, μ_B is the Bohr magneton). For strong magnetic fields the variation of ω_j with B is obtained from the Breit-Rabi formula [4].

Since spontaneous decay from state $|1\rangle$ back to $|0\rangle$ is negligible on this hyperfine transition, laser light is used to optically pump the ion into the $|0\rangle$ state via excitation of the $|1\rangle = |S_{1/2}, F = 1, m_F = 1\rangle$-$|P_{1/2}\rangle$ electric dipole transition. This laser light serves at the same time for state selective detection by collecting resonance fluorescence on this transition, and for initial Doppler cooling of the ions.

The sideband transition between states $|0\rangle$ and $|1\rangle$ can be driven by two laser light fields with appropriate detuning (Raman transition), or directly using microwave radiation. First we will consider the case when two-photon Raman transitions are implemented to cool the ion chain.

D. Simultaneous sideband cooling using optical radiation

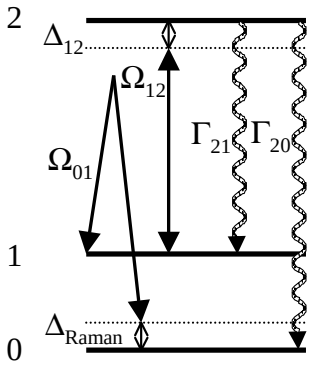

FIG. 2: Sketch of the ionic 3-level scheme that has been used for the subsequent calculations. Indicated are the relevant Rabi frequencies (symbols Ω), spontaneous decay rates (Γ), and detunings (Δ).

Sideband cooling for 10 ions with mass 171 a.m.u. has been numerically simulated by solving the optical Bloch equations for a 3-level system as depicted in Fig. 2 [25, 26]. The two counter-propagating light fields close to 369nm inducing Raman sideband transitions between levels 0 and 1 have been taken far detuned from the resonance between levels 1 and 2 such that spontaneous Raman transitions are negligible compared to the stimulated process. A third light field is tuned close to the resonance 1-2 (369 nm) and serves as a repumper into state 0. The spontaneous decay rates $\Gamma_{21} = 11 \times 2\pi$MHz and $\Gamma_{20} = 5.5 \times 2\pi$MHz have been used in these calculations. The steady state temperature has been calculated for each mode and the contributions from each ion are summed incoherently.

Fig. 3a) displays the mean vibrational quantum number $\langle n\rangle$ in the steady state of the COM mode with angular frequency $\nu_1 = 1 \times 2\pi$MHz as a function of the detuning $\Delta_{\mathrm{R}aman} = [(\omega_{R1} - \omega_{R2}) - \omega_1]$ of the Raman light fields with frequencies $\omega_{R1,R2}$ where $\Delta_{\mathrm{R}aman} = 0$ corresponds to driving the 0-1 ionic resonance. The Rabi frequency $\Omega_{12} = 100 \times 2\pi$kHz and $\Omega_{01} = 5 \times 2\pi$kHz while the detuning Δ_{12} is set to $-10 \times 2\pi$MHz. Three resonances are visible in Fig. 3a): the leftmost resonance corresponds to the first ion and occurs at $\Delta_{\mathrm{R}aman} = -\nu_1$ signifying sideband cooling of the COM mode of the whole ion chain. The cooling resonance in the middle stems from the second ion while the one on the right is caused by the third ion in the chain (compare Fig. 1 for the location of the resonances). Heating of the ions' motion occurs when the detuning $\Delta_{\mathrm{R}aman}$ is chosen such that it coincides with the blue sideband of the COM mode of the first ion.

The steady state temperature of vibrational modes characterized by ν_1, ν_2, ν_3, and ν_4 in Fig. 3b) has been calculated using the same parameters as in Fig. 3a). The resonances visible in this figure can again be identified by comparison with Fig. 1. A common resonance occurs at $\Delta_{\mathrm{R}aman} = -\nu_1$ where all four vibrational modes are cooled to low temperatures. The mean vibrational quantum number $\langle n\rangle$ of all ten axial modes around this resonance is displayed in Fig. 3c). For $\Delta_{\mathrm{R}aman} = -\nu_1$, $\langle n\rangle$ reaches its minimum for all modes. Mode number 10 displays a relatively narrow resonance and is not cooled as well as the other modes, since ion 10 participates only little in the vibrational motion of mode 10 which is reflected in the parameter S_{10}^{10} having a small magnitude. Still, mode 10 reaches a low temperature characterized by $\langle n\rangle < 10^{-3}$. Fig. 3d) displays the temperature of each mode when the detuning of the Raman beams is set close to $-\nu_1$.

The new cooling scheme introduced here works well when Raman sideband cooling is applied. In [16] it was shown that with the application of a magnetic field gradient an *effective* LDP arises that allows for coupling of internal and external dynamics even when the usual LDP is negligibly small. For instance, sideband cooling can be used even when the internal transition (here 0-1) is driven directly by long-wavelength radiation with which a small LDP is associated. In the following paragraphs sideband cooling using long wavelength radiation in the presence of a magnetic field gradient will be discussed. With the help of numerical calculations we will then investigate how effective simultaneous cooling of all vibrational modes can be when microwave instead of optical radiation is used to drive the 0-1 transition.

E. Sideband cooling using microwave fields

The recoil experienced by an ion upon absorption or emission of a microwave photon is negligible. Instead, the coupling between internal and motional degrees of freedom is due to the displacement of an ion from its equilibrium position following the absorption of a microwave photon: the magnetic field gradient modifies the mechanical potential of the state $|1\rangle$, and the two internal states experience different mechanical potentials [4, 9, 16]. In the reference frame of the harmonic oscillator, an effective LDP can be defined [9]:

$$\eta_j^{\alpha\prime} e^{i\varphi_j} \equiv \eta_j^\alpha + i\tilde{\eta}_j^\alpha \tag{6}$$

where

$$\tilde{\eta}_j^{\alpha\prime} = S_j^\alpha \frac{\partial_z \omega_0^{(j)}\, \Delta z_\alpha}{\nu_\alpha} \ , \tag{7}$$

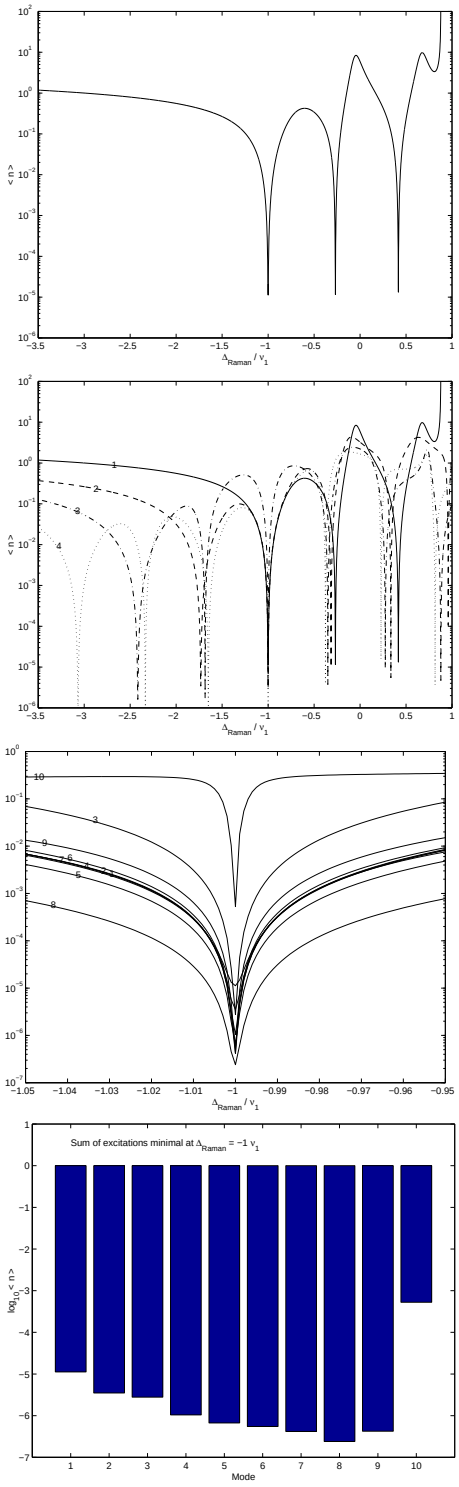

FIG. 3: a) Mean vibrational quantum number $\langle n\rangle$ as a function of the detuning Δ_{Raman} when Raman sideband cooling 10 ions. $\Omega_{12} = 100 \times 2\pi$kHz, $\Omega_{01} = 5 \times 2\pi$kHz, $\Delta_{12} = -10 \times 2\pi$MHz. a) COM Mode ($\nu_1$). b) Modes number 1 through 4. c) Modes number 1 through 10. The shape of the resonances due to modes 1, 2, 4, 6, and 7 is very similar. d) $\langle n\rangle$ in steady state for each mode at $\Delta_{Raman} = -\nu_1$. All ten vibrational modes are simultaneously cooled close to their respective ground state.

and $\partial_z\omega_0(z_j)$ is the spatial derivative of the resonance frequency with respect to z and $\Delta z_\alpha = \sqrt{\hbar/(2m\nu_\alpha)}$. Since in the microwave region η_j^α is much smaller than the second term in eq. 6, we have $\eta_j^{\alpha\prime} \approx \tilde{\eta}_j^\alpha$.

The effective linewidth of the cooling process, which sets the rate of cooling, is determined by the intensity and detuning of the repump laser operated below saturation. The recoil due to scattering of optical photons and in presence of the field gradient is represented by the Lamb-Dicke parameter λ_j^α, derived in section IV. Perturbation theory of second order in the Lamb-Dicke parameter is justified provided that $|\lambda_j^{\alpha\prime}|, |\eta_j^{\alpha\prime}| \ll 1$.

With γ being the effective linewidth of the process, and assuming the repumping laser orthogonal to the axis of the motion (so that there is no contribution to the recoil due to the absorption of laser photons) the rates derived in section IV take on the the form

$$A_\pm^\alpha = \sum_{j=1}^{N} \frac{\Omega(z_j)^2}{2\gamma}\Big[|\tilde{\eta}_j^\alpha|^2 \frac{\gamma^2}{4(\delta_j \mp \nu_\alpha)^2 + \gamma^2} + \phi|\lambda_j^\alpha|^2 \frac{\gamma^2}{4\nu_\alpha^2 + \gamma^2}\Big] \quad (8)$$

F. Simultaneous sideband cooling using microwave radiation

Fig. 4a) shows the results of numerically solving the optical Bloch equations as in section II D, that is, the steady state mean vibrational quantum number $\langle n\rangle$ of 10 axial vibrational modes of a string of 10 ions at a trap frequency $\nu_1 = 2\pi \times 1$MHz as a function of the detuning of the microwave radiation from the hyperfine transition of ion 1. The same parameters have been used as for generating Fig. 3. In Fig. 4b) the final excitation number of all vibrational modes around $\Delta_{MW} = -\nu_1$ is displayed, and Fig. 4c) shows the final mean excitation of all 10 modes at $\Delta_{MW} = -\nu_1$.

All vibrational modes are cooled close to their ground state as desired. The mode with the highest vibrational frequency ν_{10} is not cooled as well as the other modes. What is the origin of this behavior? The static field gradient applied to the trap shifts the resonance of ion 10 such that its sideband resonance $\omega_{10} - \nu_{10}$ coincides with the desired sideband resonances of the other ions, that is, ion number 10 is used to cool vibrational mode number 10. However, ion 10 hardly participates in the motion of mode 10. This is evident in the small value of $S_{10}^{10} = 0.0018$, and leads to a small effective LDP $\eta_{10}^{10\prime}$ (eq. 6) indicating the weak coupling between internal states of ion 10 and vibrational mode 10.

The small value of the coefficient S_j^α with $j = N = \alpha$ leads to a small LDP in the optical case, too, and consequently to less effective cooling of mode N than of the other modes. Despite this, using an optical Raman process still gives more effective cooling, not only of mode N but also of all other vibrational modes, than sideband

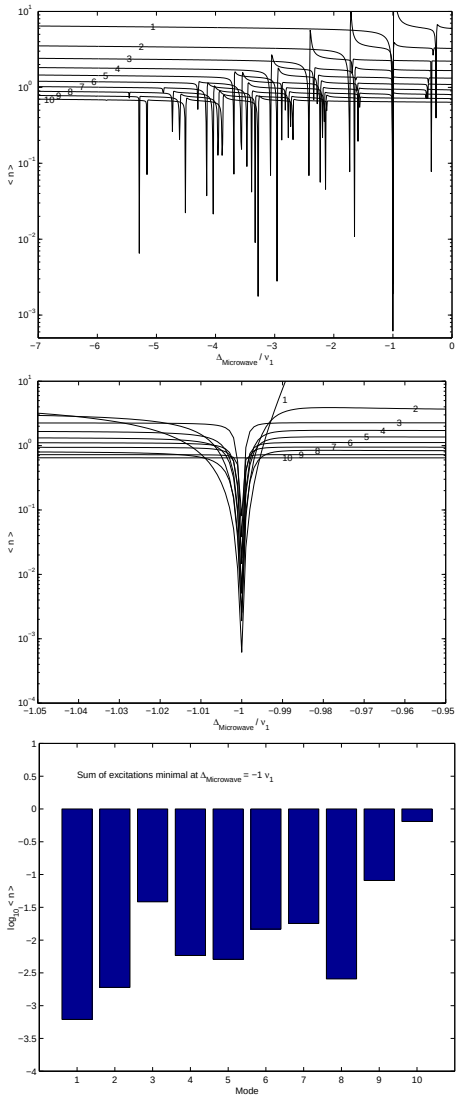

FIG. 4: a) Mean vibrational quantum number $\langle n\rangle$ as a function of the detuning $\Delta_{Microwave}$ when microwave transitions are used for sideband cooling 10 ions. $\Omega_{12} = 100 \times 2\pi$kHz, $\Omega_{01} = 5 \times 2\pi$kHz, $\Delta_{12} = -10 \times 2\pi$MHz. a) Survey spectrum of vibrational modes 1 through 10. b) Modes 1 through 10 around $\Delta_{Microwave} = -\nu_1$. c) $\langle n\rangle$ in steady state for each mode at $\Delta_{Microwave} = -\nu_1$. All ten vibrational modes are simultaneously cooled close to their respective ground state.

cooling driven by microwave radiation. This is evident from Figs. 3 and 4. The reason being that in the optical case the real part of the LDP (eq. 6) dominates and is considerably larger than the imaginary part caused by the magnetic field gradient. When using microwaves the real part of the LDP almost vanishes, due to negligible recoil imparted to the ions by microwave photons. The imaginary part can in principle be made larger by increasing the magnitude of the magnetic field gradient. However, in order to superimpose the first order red sidebands of all vibrational modes the magnetic field gradient is fixed by the distance between neighboring ions which in turn depends on the trap frequency ν_z (see also section III). When doing quantum logic, or other operations that require large coupling between internal and external states, then the magnetic field gradient can be increased such that the effective LDP reaches the desired magnitude [16].

The simultaneous cooling of all vibrational modes can be combined with quantum logic operations using mw radiation when alternating between periods of cooling (small field gradient) and periods of doing quantum logic (large field gradient). After cooling, the field gradient has to be ramped up to the value needed for individual addressing of qubits. The gradient has to be varied adiabatically, i.e. on a time scale large compared to the period of vibrational motion, in order not to excite the ion string's motion.

With heating rates of the order 100ms as have been observed experimentally [27], many quantum logic operations can be carried out before decoherence of motional states sets in.

III. EXPERIMENTAL CONSIDERATIONS

In this section we discuss how the field gradients for simultaneous sideband cooling can be generated and how cooling is affected by imperfectly superimposing the red motional sidebands of different vibrational modes. In order to demonstrate the feasibility of the proposed scheme it is sufficient to restrict the discussion to very simple arrangements of magnetic field generating coils.

If the vibrational resonances and the ions were equally spaced in frequency and position space, respectively, then a constant field gradient, appropriately chosen, could make all N modes overlap and let them be cooled at the same time. Since $\nu_\alpha - \nu_{\alpha-1}$ decreases monotonically with growing α and the ions' mutual distances vary with j, the magnetic field gradient has to be adjusted along the $z-$axis. The field gradient needed to shift the ions' resonances by the desired amount is obtained from

$$\left.\frac{\partial B}{\partial z}\right|_{(z_j+z_{j-1})/2} \approx \frac{B(z_j) - B(z_{j-1})}{z_j - z_{j-1}} \stackrel{!}{=} \frac{\upsilon_j - \upsilon_{j-1}}{\zeta_j - \zeta_{j-1}} \zeta_0 \nu_1 \frac{\hbar}{\mu_B},\ j = 2, \ldots, N \quad (9)$$

where $\zeta_j \equiv z_j/\zeta_0$ is the scaled equilibrium position of ion j, and υ_j is the square root of the $j-$th eigenvalue of the dynamical matrix. Eq. 9 describes the situation for moderate magnetic fields (the Zeeman energy is much smaller than the hyperfine splitting), such that $\partial_z\omega_j = 1/2\ g_J\mu_B\partial_z B$ with $g_J \approx g_s = 2$ (state $|0\rangle$ does not depend on B). As an example, we consider a string of $N = 10$ ^{171}Yb$^+$ions in a trap characterized by $\nu_z = 1 \times 2\pi$MHz ($\zeta_0 = 2.7\mu$m).

The markers in Fig. 5 indicate the values of the required field gradient according to eq. 9 whereas the solid

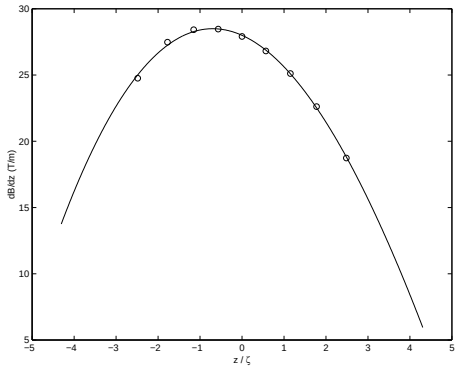

FIG. 5: Required magnetic field gradient to superpose the motional red sidebands of ten $^{171}Yb^{+}$ ions (markers) in a trap characterized by a COM frequency $\nu_1 = 1 \times 2\pi$MHz and calculated field gradient (solid line) produced by three single wire windings (see text).

line shows the gradient generated by 3 single windings of diameter 100μm, located at $z = -100, 50$, and 100 μm $\approx 36\zeta_0$, respectively . Running the currents -5.33A, -6.46A, and 4.29A through these coils produces the desired field gradient at the location of the ions. Micro electromagnets with dimensions of a few tens of micrometers and smaller are now routinely being used in experiments where neutral atoms are trapped and manipulated [28, 29]. Current densities up to 10^8A/cm^2 have been achieved in such experiments. A current density more than two orders of magnitude less than was achieved in atom trapping experiments would suffice in the above mentioned example [30].

This configuration of magnetic field coils shall serve as an example to illustrate the feasibility of the proposed cooling scheme in what follows. It will be shown that with a few current carrying elements in such a simple arrangement one may obtain good results when simultaneously sideband cooling all axial modes. More sophisticated structures for generating the magnetic field gradients can of course be employed, making use of more coils, different diameters, variable currents, or completely different configurations of current carrying structures

Fig. 6 shows the location (relative to ω_1) of the $j-$th red sideband resonance (corresponding to the $j-$th vibrational mode of the ion string) of the $j-$th ion, $\omega_j - \nu_j$ with $j = 1, \ldots, 10$. The height of the bars in Fig. 6 indicates the strength of the coupling between the driving radiation and the respective sideband transition relative to the COM sideband of ion number 1. The relevant coupling parameter is the LDP. For optical transitions $|\eta_j^{j\prime}|/|\eta_1^{1\prime}| \approx |\eta_j^j|/|\eta_1^1| = S_j^j \nu_j^{-1/2}/S_1^1\nu_1^{-1/2}$ whereas microwave transitions between the hyperfine states are characterized by $|\eta_j^{j\prime}|/|\eta_1^{1\prime}| \approx S_j^j\nu_j^{-3/2}/S_1^1\nu_1^{-3/2}$ (compare eq. 6 and eq. 2). The ratio of these parameters is much smaller for the highest vibrational mode ($j = 10$) than for other modes, since ion 10 is only slightly displaced

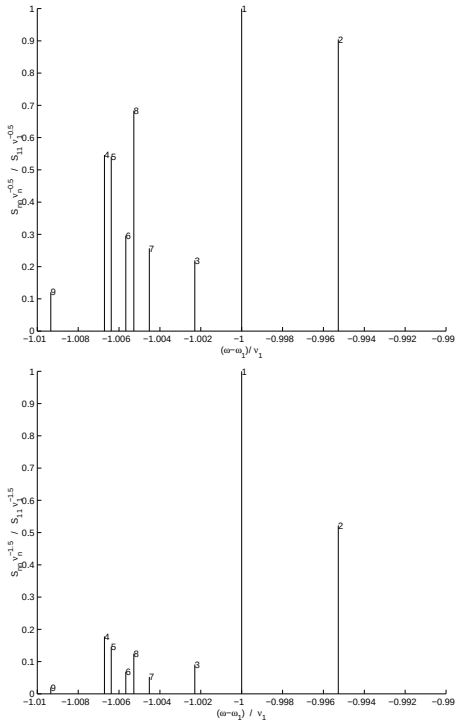

FIG. 6: The positions of first order sideband resonances corresponding to 10 axial vibrational modes of 10 $^{171}Yb^{+}$ ions in an ion-trap with the field gradient shown in Fig. 5. The height of the bars indicates the transition probability of a given sideband relative to the first sideband of the COM mode. a) Raman transition. The relative transition probability is proportional to $|\eta_j^{j\prime}|/|\eta_1^{1\prime}| \approx S_j^j\nu_j^{-(1/2)}$ b) Microwave transition. The transition probability is proportional to $|\eta_j^{j\prime}|/|\eta_1^{1\prime}| \approx S_j^j\nu_j^{-(3/2)}$

from its equilibrium position when mode 10 is excited.

Ideally, all 10 resonances would be superimposed for optimal cooling. The resonances shown in Fig. 6a) result from the field gradient calculated using the configuration described above for a trap frequency $\nu_1 = 1 \times 2\pi$1MHz. Even though they do not fall on top of each other, these resonances all lie within a frequency interval of about $0.015 \times \nu_1 = 15$kHz.

In Fig. 7a) the steady state vibrational excitation $\langle n\rangle$ of a string of 10 $^{171}Yb^{+}$ions at $\nu_z = 1 \times 2\pi$MHz is displayed as a function of the detuning of the Raman beams relative to the resonance frequency ω_1 of ion 1. The Rabi frequencies and detuning, too, are the same as have been used to generate Fig. 3. However, the field gradient that shifts the ions' resonances is not assumed ideal as in Fig. 3, instead the one generated by three single windings as described above (Fig. 5) has been used. Despite the imperfect superposition of the cooling resonances, low temperatures of all modes close to their ground state can be achieved as can be seen in Fig. 7b). Here, the value of $\langle n\rangle$ for each mode has been plotted at that detuning $\Delta_{Raman} = -1.008\nu_1$ where the sum of all

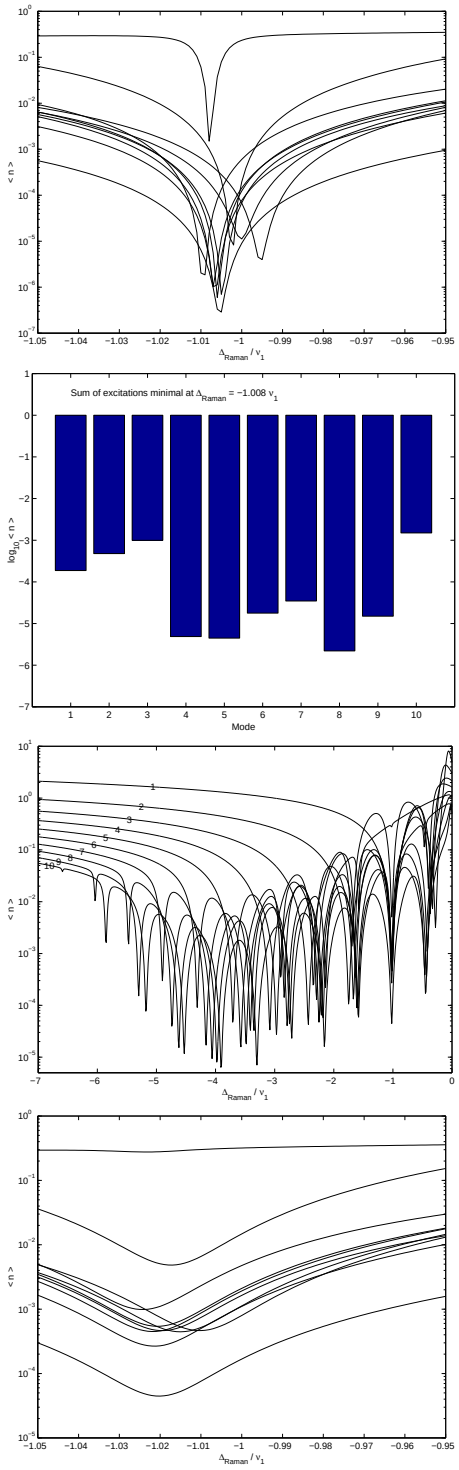

FIG. 7: a) Steady state vibrational excitation for Raman cooling. The field gradient used here is the one displayed in Fig. 5. a) $\langle n\rangle$ as a function of the detuning Δ_{Raman}. Same parameters as in Fig. 3 b) $\langle n\rangle$ for each mode at that detuning where the sum of the mean vibrational quantum numbers of all ten modes is minimal. c) Wide range scan of Δ_{Raman} with all parameters unchanged except $\Omega_{12} = 1 \times 2\pi$MHz. d) Similar to a) except $\Omega_{12} = 1 \times 2\pi$MHz.

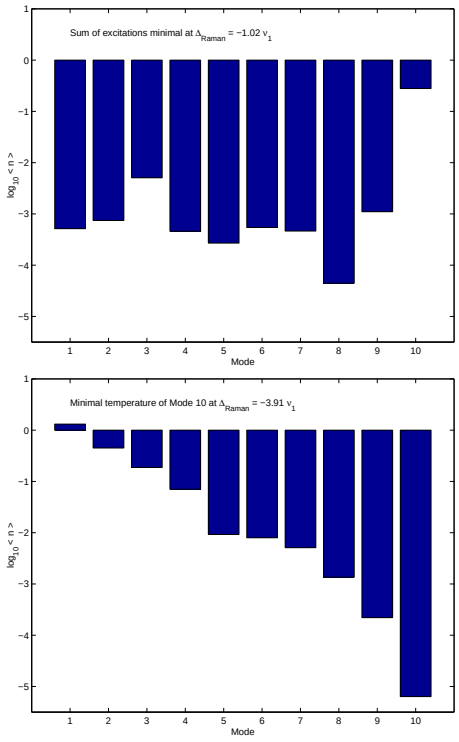

FIG. 8: a) Similar to Fig 7b except $\Omega_{12} = 1 \times 2\pi$MHz. b) $\langle n\rangle$ for each mode at that detuning where one of the modes (here mode 10) reaches the absolute minimum.

excitations is minimal. Fig. 7c displays the excitation of each mode over a wide range of the detuning such that all first order (in $\eta_j^{\alpha\prime}$) resonances corresponding to red sidebands are visible. Here, the Rabi frequency Ω_{12} of the repump laser has been increased to $1 \times 2\pi$MHz as compared to $100 \times 2\pi$kHz in the previous figures. This results in broader resonances as is evident in Fig. 7d and higher final temperatures. Setting the Raman beams to a desired detuning and keeping their relative detuning constant as a function of time is done by translating into the optical domain the microwave or radio frequency that characterizes the splitting of states 0 and 1 (using, for example, acousto or electro-optic modulators). Microwave or rf signals can be controlled with high precision and display low enough drift to ensure efficient cooling. The requirements regarding both the precision of adjustment and the drift of the frequency source are further relaxed if a large enough intensity of the repump laser is employed (as in Fig. 7c and d). Fig. 8a shows $\langle n\rangle$ for each mode similar to 7b, however with $\Omega_{12} = 1 \times 2\pi$MHz.

Efficient cooling does not only occur around the resonance $\Delta_{Raman} = -\nu_1$ but also at other values of Δ_{Raman} as can be seen in Fig. 7c. As an example, Fig. 8b shows $\langle n\rangle$ of all modes at that detuning where one of the modes reaches the absolute minimum when Δ_{Raman} is varied. This is the case for mode 10 at $\Delta_{Raman} = -3.91\nu_1$. At this resonance the red sideband of the 5th ion (the cen-

ter ion) corresponding to the 10th mode is driven by the Raman beams. All other vibrational modes are still reasonably cold.

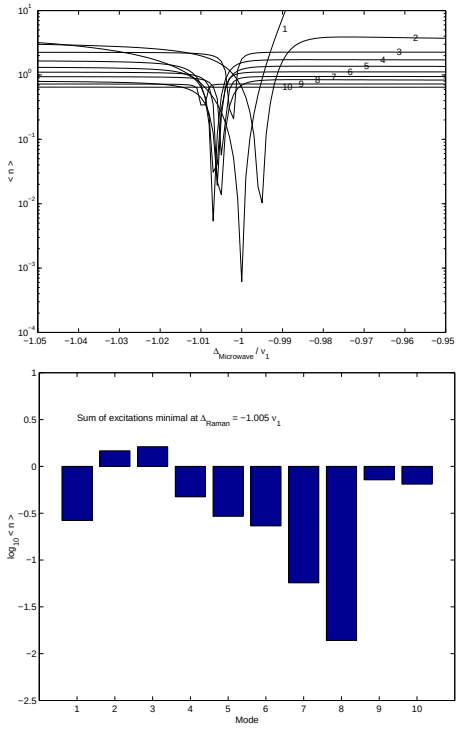

FIG. 9: a) Steady state vibrational excitation as a function of the detuning Δ_{MW} when the sideband transition is driven by microwave radiation. The field gradient used here is the one displayed in Fig. 5. Rabi frequencies and detuning are the same as in Fig. 4. a) $\langle n\rangle$ as a function of the detuning $\Delta_{Microwave}$. b) $\langle n\rangle$ for each mode at that detuning where the sum of the mean vibrational quantum numbers of all ten modes is minimal.

The data in Fig. 9a and b have been generated with the same parameters as in Fig. 4, that is microwave radiation is used to drive the sidebands of the hyperfine transition 0-1. Since the effective LDP is much smaller as compared to the optical case (see the discussion in section II F), the resonances pertaining to the vibrational modes are narrower, and the deviation of the real magnetic field gradient from its ideal shape affects simultaneous cooling more than in the optical case. Still, at $\Delta_{MW} = -1.005\nu_1$ all modes are reasonably cold. Already with such a basic setup of 3 coils generating the magnetic field gradient effective simultaneous cooling of axial vibrational modes can be implemented even using microwave radiation. Using more than 3 magnet coils will allow to shape the gradient more accurately and thus allow for an even better superposition of the sidebands.

For the cooling scheme introduced here to work, the field gradient has to vary in the axial direction and it remains to be shown in what follows that this variation is compatible with the neglect of higher-order terms in q_j and $\partial_z B$ which is justified as long as $|q_j^2\partial_z^2 B| \ll |q_j\partial_z B|$. Using 9 and $q_j \approx \Delta z = \sqrt{\hbar/2m\nu_1}$ this condition can be written as

$$\frac{2}{\zeta_{j+1}-\zeta_{j-1}}\left|\frac{(\lambda_{j+1}-\lambda_j)(\zeta_j-\zeta_{j-1})}{(\lambda_{j+1}-\lambda_j)(\zeta_j-\zeta_{j-1})}-1\right| \ll \frac{\zeta_0}{\Delta z}\ . \quad (10)$$

Considering the region where the second derivative of the field is maximal ($n = 9$) and inserting numbers into 10 gives for ten ions $0.24 \ll 2.9\times 10^3 (m/\nu_1)^{\frac{1}{6}}$. The right-hand side of this inequality is dimensionless if m is inserted in amu and yields ≈ 500 for $^{171}Yb^+$ions and $\nu_1 = 1\times 2\pi$MHz. Since the typical distance, ζ_0 over which the gradient has to vary is much larger than the range of motion, $q_j \approx \Delta z$ of an individual ion, the approximation of a linear field gradient is a good one.

IV. THEORETICAL MODEL

In this section, we introduce the basic equations, from which the rates (8) have been derived.
We consider a chain consisting of N identical ions aligned along the z-axis, and in presence of a magnetic field $B(z)$. The internal electronic states of each ion which are relevant for the dynamics are the stable states $|0\rangle$ and $|1\rangle$ and the excited state $|r\rangle$. The transitions $|0\rangle \to |1\rangle$, $|1\rangle \to |2\rangle$ are respectively a magnetic and an optical dipole transition. We assume that the magnetic moments of $|0\rangle$ and $|2\rangle$ are zero, while $|1\rangle$ has magnetic moment μ, and its energy with respect to $|0\rangle$ is therefore shifted proportionally to the field, $\hbar\omega_0(z) \propto |B(z)|$. The Hamiltonian describing the internal degrees of freedom has the form:

$$H_{\rm int} = \hbar\sum_j \left(\omega_0(z_j)|1\rangle_j\langle 1| + \omega_2|2\rangle_j\langle 2|\right) \quad (11)$$

where the index j labels the ions along the chain.

The collective excitations of the chain are described by the eigenmodes at frequency $\nu_1,\ldots,\nu_N$, which are independent of the internal states. Denoting with Q_α, P_α the normal coordinates and conjugate momenta of the oscillator at frequency ν_α, the Hamiltonian for the external degrees of freedom then has the form

$$H_{\rm mec} = \frac{1}{2m}\sum_{\alpha=1}^{N} P_\alpha^2 + \frac{m}{2}\sum_{\alpha=1}^{N}\nu_\alpha^2\left[Q_\alpha + \frac{\hbar}{2m\nu_\alpha^2}\sum_j \left.\frac{\partial\omega_j}{\partial z_j}\right|_{z_{0,j}} |1\rangle\langle 1|S_j^\alpha\right]^2 \quad (12)$$

where we have neglected the higher spatial derivatives of the magnetic field, as well as higher powers of the derivative of the field.
Thus, the coupling of the excited state $|1\rangle$ to a spatially varying magnetic field shifts the center of the oscillators for the ions being in the excited state.
The ions magnetic dipole transition is driven by a microwave field at Rabi frequency Ω_μ and frequency ω_μ.

The interaction reads

$$W_\mu = \sum_j \frac{\hbar\Omega_\mu}{2}\left[|1\rangle_j\langle 0|\mathrm{e}^{-\mathrm{i}(\omega_\mu t - k_\mu z_j+\phi)} + \mathrm{H.c.}\right] \quad (13)$$

where $\vec{k_\mu}$ is the wave vector of the field, and where we have assumed that the spatial phase of the field is constant over the extent of the ionic wave packet. This is justified, since the displacement of an ion from its equilibrium position is many orders of magnitude smaller than the microwave wavelength.
The transition $|1\rangle \rightarrow |2\rangle$ is driven under saturation by a laser at Rabi frequency Ω_{12}, frequency ω_{12}, and wave vector $\vec{k}$. The interaction term reads:

$$W_{12} = \sum_j \frac{\hbar\Omega_{12}}{2}\left[|2\rangle_j\langle 1|\mathrm{e}^{-\mathrm{i}(\omega_{12}t - kz_j+\phi)}\mathrm{e}^{\mathrm{i}kq_j} + \mathrm{H.c.}\right] \quad (14)$$

where q_j is the displacement of the ion j from the classical equilibrium position z_j.
The master equation for the density matrix ρ, describing the internal and external degrees of the ions, reads:

$$\frac{\partial}{\partial t}\rho = \frac{1}{\mathrm{i}\hbar}[H,\rho] + \mathcal{L}\rho \quad (15)$$

where $H = H_0 + H_{\mathrm{mec}} + W_\mu + W_L$, and $\mathcal{L}\rho$ is the Liouvillian describing the spontaneous emission processes, i.e. the decay from the state $|2\rangle$ into the states $|0\rangle$ and $|1\rangle$ at rates Γ_{20}, Γ_{21}, respectively, where $\gamma = \Gamma_{20} + \Gamma_{21}$ is the total decay rate. Here, we assume $\Gamma_{20}/\Gamma_{21} \gg 1$ [31].

***In den numerischen Rechnungen unterscheiden sich die beiden Raten um einen Faktor zwei. Kann das auch hier bercksichtigt werden?

The Liouvillian aquires then the form:

$$\mathcal{L}\rho = \gamma\sum_j\Big[-\frac{1}{2}|2\rangle_j\langle 2|\rho - \frac{1}{2}\rho|2\rangle_j\langle 2| + \int_1^1 \mathrm{d}\cos\theta\mathcal{N}(\cos\theta)\mathrm{e}^{\mathrm{i}k\cos\theta q_j}|0\rangle_j\langle 2|\rho|2\rangle_j\langle 0|\mathrm{e}^{-\mathrm{i}k\cos\theta q_j}\Big] \quad (16)$$

***Integrationsgrenzen?

In order to study the dynamics, it is convenient to move to the inertial frames rotating at the field frequencies. Moreover, we apply the unitary transformation

$$U = \exp\left[-\mathrm{i}\sum_\alpha\left(\frac{1}{2m\nu_\alpha^2}\sum_j\frac{\partial\omega_{0,j}}{\partial z_j}\Big|_{z_{0,n}}|1\rangle_j\langle 1|S_j^\alpha\right)P_\alpha\right] \quad (17)$$

We denote with $\tilde{\rho}$ the density matrix in the new reference frame. The master equation now reads

$$\frac{\partial}{\partial t}\tilde{\rho} = \frac{1}{\mathrm{i}\hbar}[\tilde{H},\tilde{\rho}] + \mathcal{L}\tilde{\rho} \quad (18)$$

where $\tilde{H} = \tilde{H}_0 + \tilde{H}_{\mathrm{mec}} + \tilde{W}_\mu + \tilde{W}_{12}$, and the individual terms now have the form:

$$\tilde{H}_0 = \hbar\sum_j[\delta(z_j)|0\rangle_j\langle 0| + \Delta(z_j)|2\rangle\langle 2|] \quad (19)$$

with $\delta(z_j) = \omega_\mu - \omega_0(z_j)$ and $\Delta(z_j) = \omega_2 - \omega_0(z_j) - \omega_{12}$. The mechanical energy is now

$$\begin{aligned}\tilde{H}_{\mathrm{mec}} &= \frac{1}{2m}\sum_{\alpha=1}^N P_\alpha^2 + \frac{m}{2}\sum_{\alpha=1}^N\nu_\alpha^2 Q_\alpha^2 \\ &= \sum_\alpha\hbar\nu_\alpha\left(a_\alpha^\dagger a_\alpha + \frac{1}{2}\right)\end{aligned} \quad (20)$$

where $a_\alpha^\dagger$, a_α have been introduced, the creation and annihilation operator, respectively of a quantum of energy $\hbar\nu_\alpha$. The interaction term with the microwave reads

$$\tilde{W}_\mu = \sum_j\frac{\hbar\Omega_\mu}{2}\left[|1\rangle_j\langle 0|\mathrm{e}^{\mathrm{i}(k_\mu z_j-\phi)}\mathrm{e}^{-\mathrm{i}\sum_\alpha\tilde{k}_j^\alpha P_\alpha} + \mathrm{H.c}\right] \quad (21)$$

where

$$\tilde{k}_j^\alpha = \frac{\partial\omega_{0,j}}{\partial z_j}\frac{1}{2m\nu_\alpha^2}S_j^\alpha \quad (22)$$

Thus, the excitation between two states, where the mechanical potential in one is shifted with respect to the other, corresponds to an effective recoil, here described by the effective Lamb-Dicke parameter $\tilde{\eta}_j^\alpha = \tilde{k}_j^\alpha\sqrt{\hbar m\nu_\alpha/2}$. Finally, the laser-interaction has the form

$$\tilde{W}_{12} = \sum_j\frac{\hbar\Omega_{12}}{2}\left[|2\rangle_j\langle 1|\mathrm{e}^{\mathrm{i}\chi_j}\mathrm{e}^{\mathrm{i}\sum_\alpha(\lambda_\alpha a_\alpha^\dagger + \lambda_\alpha^* a_\alpha)} + \mathrm{H.c.}\right] \quad (23)$$

where χ_j is a constant phase, that depends on the position

$$\chi_j = kz_j - \phi - \hbar k\sum_\alpha\tilde{k}_j^\alpha S_j^\alpha/2 \quad (24)$$

and λ_α is an effective Lamb-Dicke parameter, which originates from the recoils due to the emission of an optical photon and due to the force exerted on the atom by making a transition from a potential to a shifted one,

***In welcher Beziehung steht λ_alpha zu η_j^α (eq. **??**)?.

$$\lambda_\alpha = -\tilde{k}_j^\alpha\sqrt{\frac{\hbar m\nu_\alpha}{2}} + kS_j^\alpha\sqrt{\frac{\hbar}{2m\nu_\alpha}} \quad (25)$$

In the Lamb-Dicke regime, which here must apply for all recoils, and for low saturations on the optical dipole transition, perturbation theory may be applied. Following standard procedures [24], we obtained a rate equation as in (3), which rates have the form (8).

V. CONCLUSIONS

If more than two ions at a time are to be used for experiments that require control of the motional states of the ions, for example quantum logic operations, it becomes difficult to cool all vibrational modes sufficiently. We

have introduced a new scheme for cooling the vibrational motion of ions in a linear trap configuration. All axial vibrational modes are simultaneously cooled close to their ground state by superimposing the red motional sidebands in the absorption spectrum of different ions such that each sideband, corresponding to a particular vibrational mode, is excited when driving an internal transition of the ions with radiation at a single frequency. The superposition of all first order (in the Lamb-Dicke parameter) red sidebands is achieved by applying a magnetic field gradient along the trap axis shifting the internal ionic resonances individually by a desired amount.

An analytic treatment of sideband cooling of a string of ions in the presence of a magnetic field gradient has been given and an explicit expression for the heating and cooling rates has been derived.

Numerical studies have shown that simultaneously Raman cooling all axial modes with this new scheme is effective for realistic sets of parameters characterizing the driving radiation and the ion trap. These studies have also revealed that using microwave radiation to drive the sideband transition is not as effective, due to the relatively small field gradient required to superimpose the sidebands, but still sufficiently low temperatures can be reached.

When using microwave radiation the photon recoil upon absorption or emission of radiation possibly (de-)exciting the vibrational motion of the ion string is negligible. Instead, the displacement of the equilibrium position of an ion in a magnetic field gradient conditioned on its internal state leads to a coupling between internal and external dynamics. The strength of this coupling is described by a new effective Lamb-Dicke parameter that is proportional to the magnetic field gradient. For the newly introduced cooling scheme, the field gradient is used to superimpose the motional sidebands, and thus can not be freely chosen (it is determined by the mutual distance between the ions and other restraints). The relatively small value of the gradient required for simultaneous cooling of all axial modes makes the coupling between internal and external states less effective. This is reflected by a small effective LDP in the microwave regime.

When implementing quantum logic operations using microwave radiation it is desirable to have stronger coupling between internal and external states, that is, a larger field gradient. Also, it may not be advantageous to address all motional sidebands with a single frequency for conditional quantum dynamics with several ions. Therefore it is useful to ramp up the field gradient to a value where all coincidences between internal and motional resonances are removed [16] after initial cooling of all vibrational modes. This should be done fast enough not to allow for appreciable heating of the ion string, for example, by stray fields, and slow enough not to excite vibrational modes in the process. A lower limit for the time it takes to ramp up the gradient seems to be $2\pi/\nu_1$ and the upper limit is set by the time needed to heat up the string of ions. The latter is determined by a particular trap setup and has been measured to take as long as 100ms [27]. Heating of high vibrational modes is expected to be even slower, since for the excitation of differential motion of the ions, fields are required that reverse sign over a distance comparable to the inter ionic spacing.

If usual schemes employing *optical* radiation for manipulating internal and external degrees of freedom of trapped ions are used, then the field gradient maybe turned off adiabatically after cooling the vibrational modes.

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